Harmonization of Friction Measuring Devices Using Robust Regression Methods

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Outline

- What is harmonization of devices?
  - ✓ Measurement conversion
  - ✓ Conversion accuracy
- The best case scenario
  - ✓ Perfectly identical devices
- Failure of Linear regression
  - ✓ Ideal (almost utopian) conditions
  - ✓ Playing a trick on you
  - ✓ Biased and inconsistent estimate

Outline

- Measurement error models
  - ✓ Orthogonal regression
  - ✓ The general model
- Practical results
  - ✓ Repeated measurements from the same device (locked wheel tester)
  - ✓ Simulation of two devices
  - ✓ Measurements from two different devices (locked wheel testers)

Harmonization

- ASTM
  - ✓ “the adjustment of the outputs of different devices used for the measurement of a specific phenomenon so that all devices report the same value”
- Does not consider agreement
  - ✓ Confidence between adjusted outputs
  - ✓ Limits of agreement (LOA)

Perfectly Identical Devices

- This is the best we can hope for
- It means:
  - ✓ If we have two devices and obtain two sets of measurements (runs) from device 1, this is the same as:
  - ✓ Obtaining two sets of measurements from device 2, or
  - ✓ Obtaining two set of measurements each from one of the devices

Failure of Linear Regression

Ideal Conditions

- Source of errors restricted to independent identically distributed normal error
- The measurements

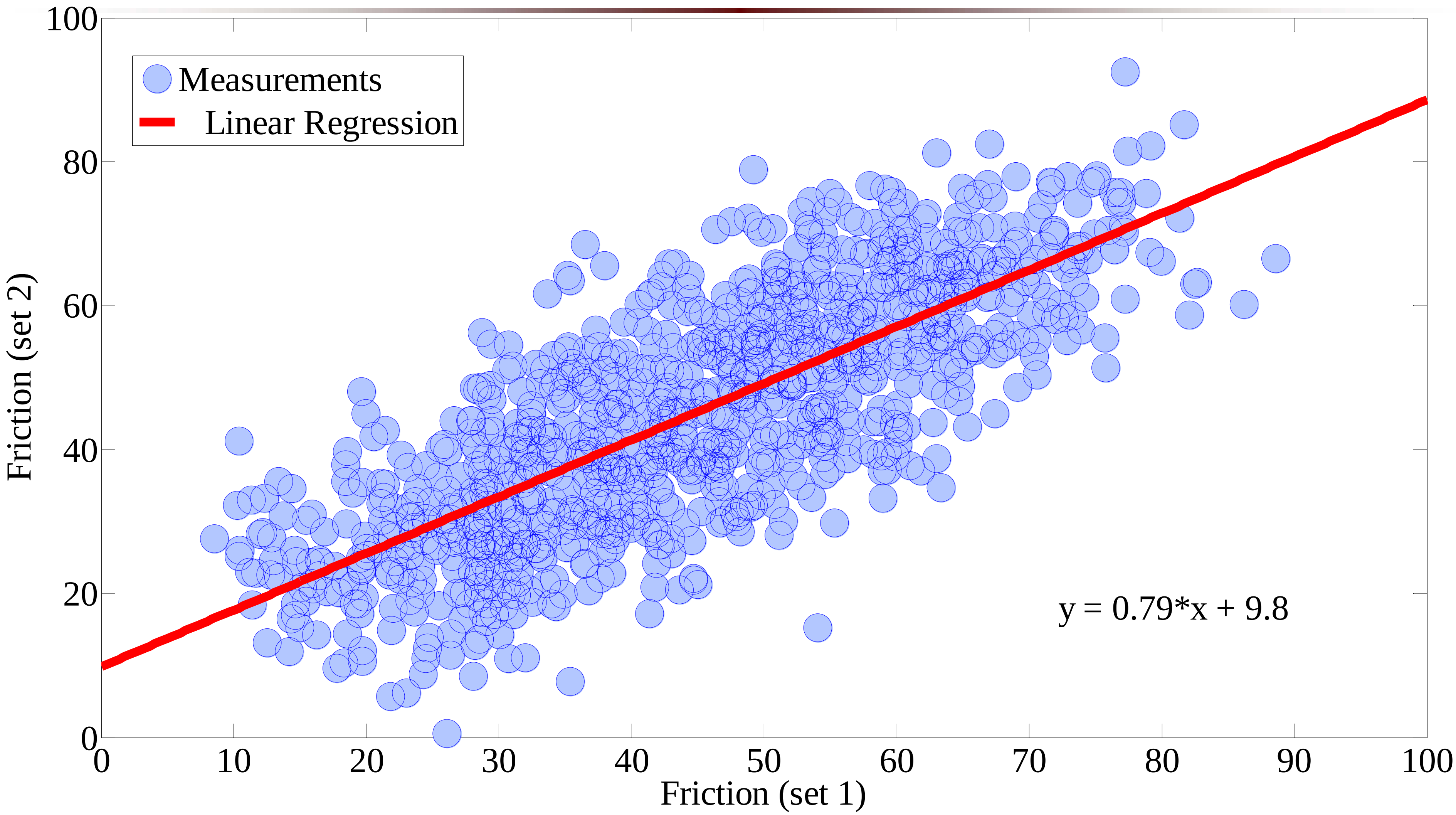
$$y_i = f_i + \varepsilon_i$$

  - ✓ y_i = measurement (with error)
  - ✓ f_i = true value
  - ✓ ε_i = error which is $N(0,\sigma^2)$

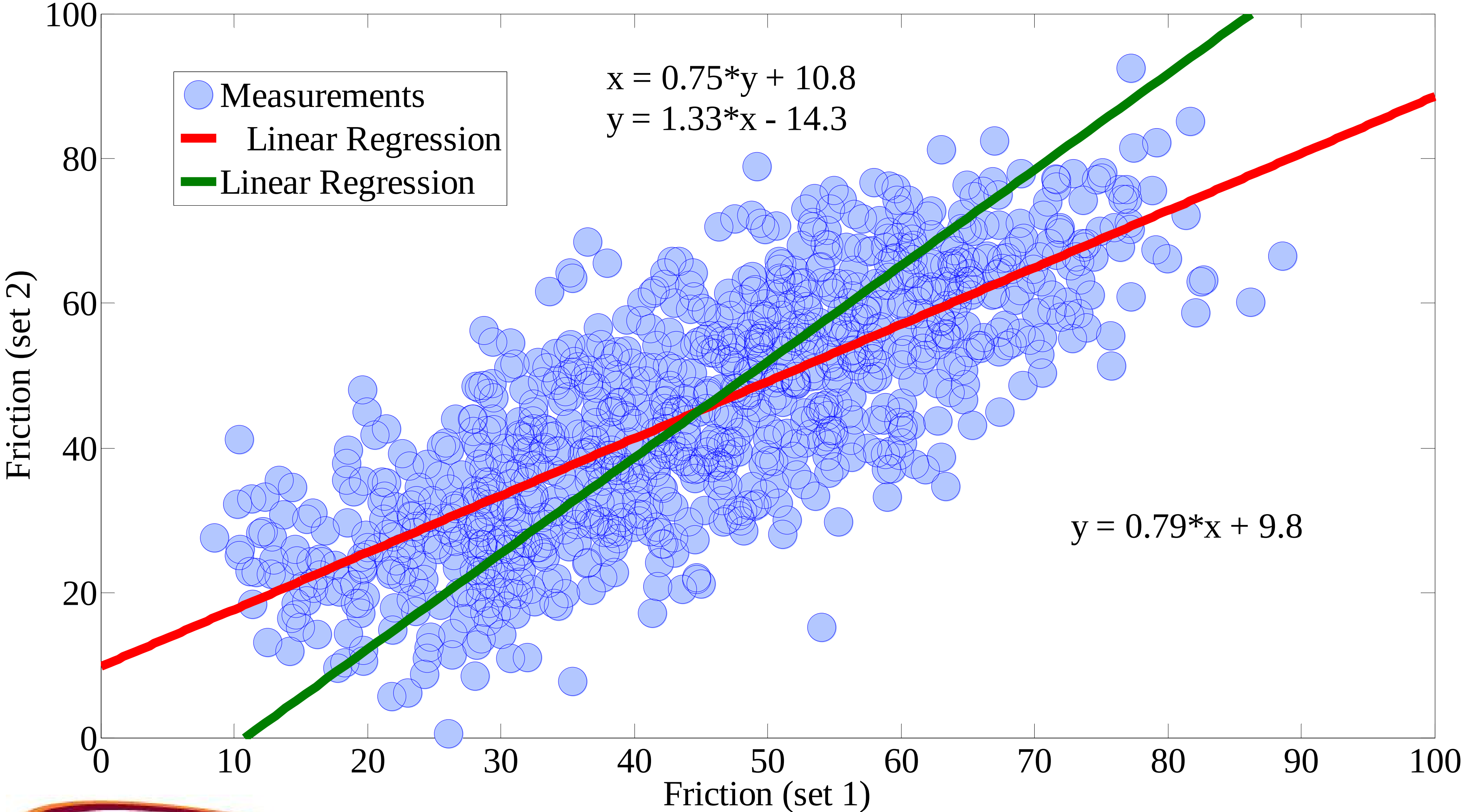
Playing a Trick on You

- You own two locked wheel skid testers
- You ask the operator to test the same pavement sections with the two devices (1 test each)
- The operator, being lazy, performs the two tests with device 1
- Testing conditions were ideal: nothing affected results other than random error
- To compare the devices you do linear regression and find…

Biased Estimate



Inconsistent Estimate



Reasons

- **Ordinary regression assumes:**
  - ✓ **Only one of the variables contains error**
  - ✓ **x variable is measured "without" error**
- **When both variables contain error:**
  - ✓ **Ordinary regression (least squares) is Biased**
  - ✓ **Bias depends on amount of error compared to range of measurement**
  - ✓ **More error more bias**

Ordinary Regression Summary

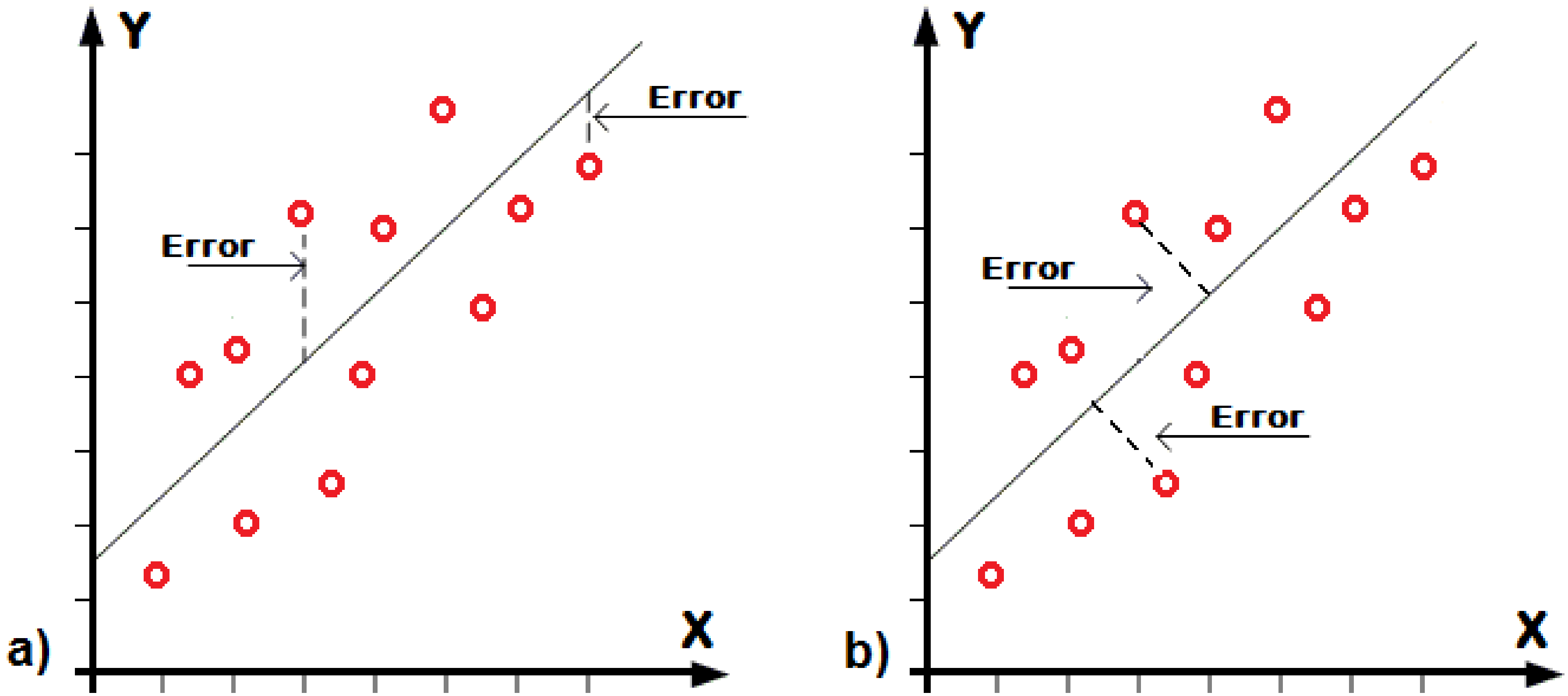
- Biased estimate
- Inconsistent estimate
- This occurs in the most simple case:
  - ✓ Error normally distributed and no other factors affecting measurement
  - ✓ Real measurements are more complicated
- Things can get worse:
  - ✓ Speed correction models not perfect: introduce MORE error
  - ✓ Texture models introduce MORE error

Measurement Error Model

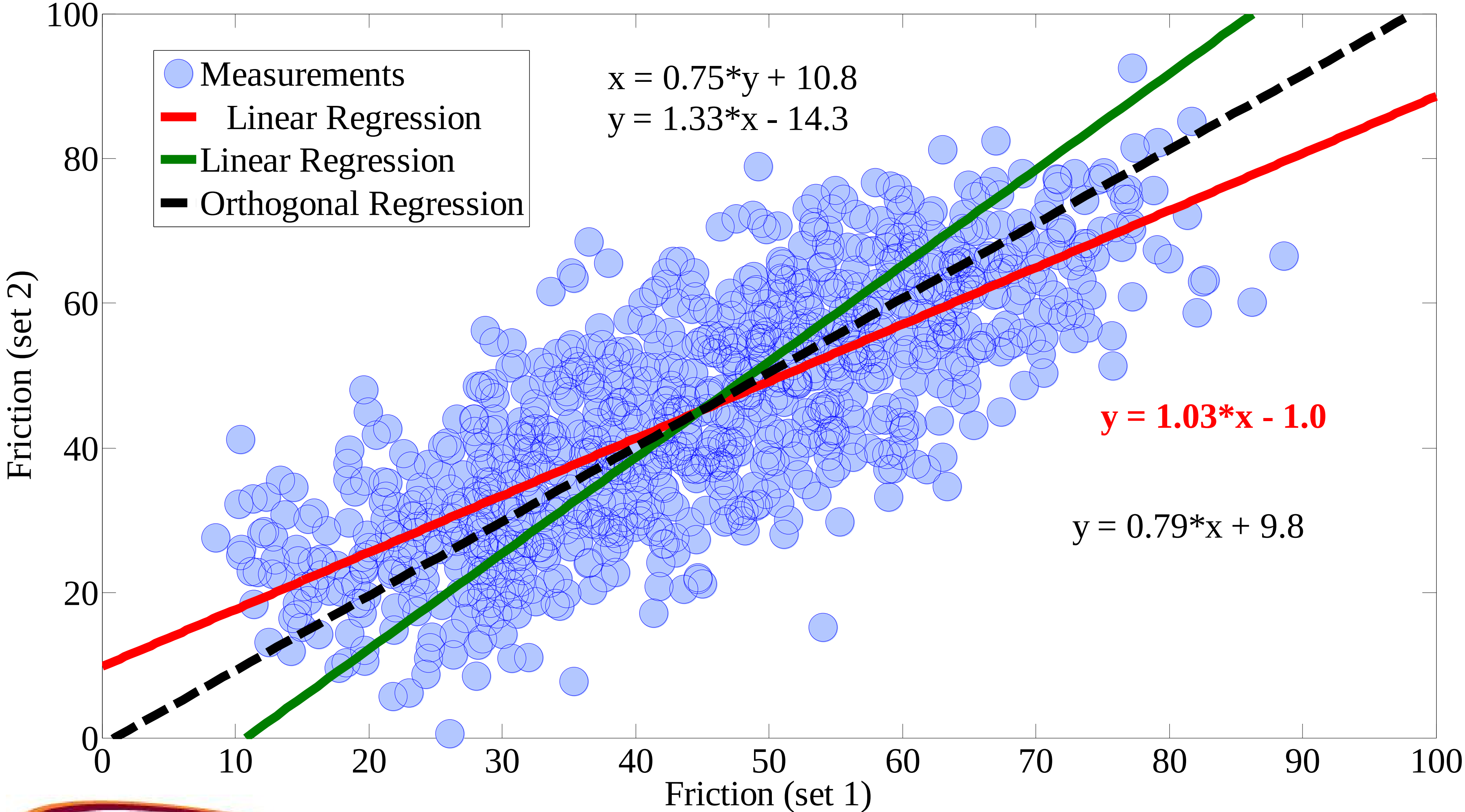
Measurement Error Model

- Take into account errors in both variables
- Simplest case: orthogonal regression
- Gives an “unbiased” estimate

Orthogonal Regression



Orthogonal Regression



Postulating the Model

- The best case only considers measurement error
- Real measurements:
  - ✓ Measurement error (repeatability)
  - ✓ Model/device error:
    - device estimates the "true" friction
    - Independent of measurement error
    - Experimental evidence of device error: between devices error

Postulating the Model

$$Y = a_Y X + b_Y + \varepsilon_Y \qquad \psi = Y + r_Y$$

- ***Y*** **= device estimate (with no repeatability error)**
- ***X*** **= "true" friction which is not known**
- **a_y and b_y = model parameters**
- **ε_Y = model error**
- **ψ = device measurement**
- **r_Y = repeatability error**

$$Z = a_Z X + b_Z + \varepsilon_Z$$

$$\zeta = Z + r_Z$$

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$$\zeta = Z + r_Z$$

Harmonization

$$Y = \frac{a_Y}{a_Z} Z + \left(b_Y - \frac{a_Y}{a_Z} b_z \right) + \varepsilon_Y + \frac{a_Y}{a_Z} \varepsilon_z = AZ + B + \mathrm{E}$$

$$\hat{A} = \frac{s_\psi^2 - \lambda\, s_\zeta^2 + \sqrt{\left(s_\psi^2 - \lambda\, s_\zeta^2\right)^2 + 4\lambda s_{\zeta\psi}^2}}{2 s_{\zeta\psi}}$$

$$\hat{B} = \overline{Y} - \hat{A}\overline{Z}$$

Harmonization

$$s_{\psi}^{2} = \mathrm{var}(\psi)$$

$$s_{\zeta\psi} = \mathrm{cov}(\zeta, \psi)$$

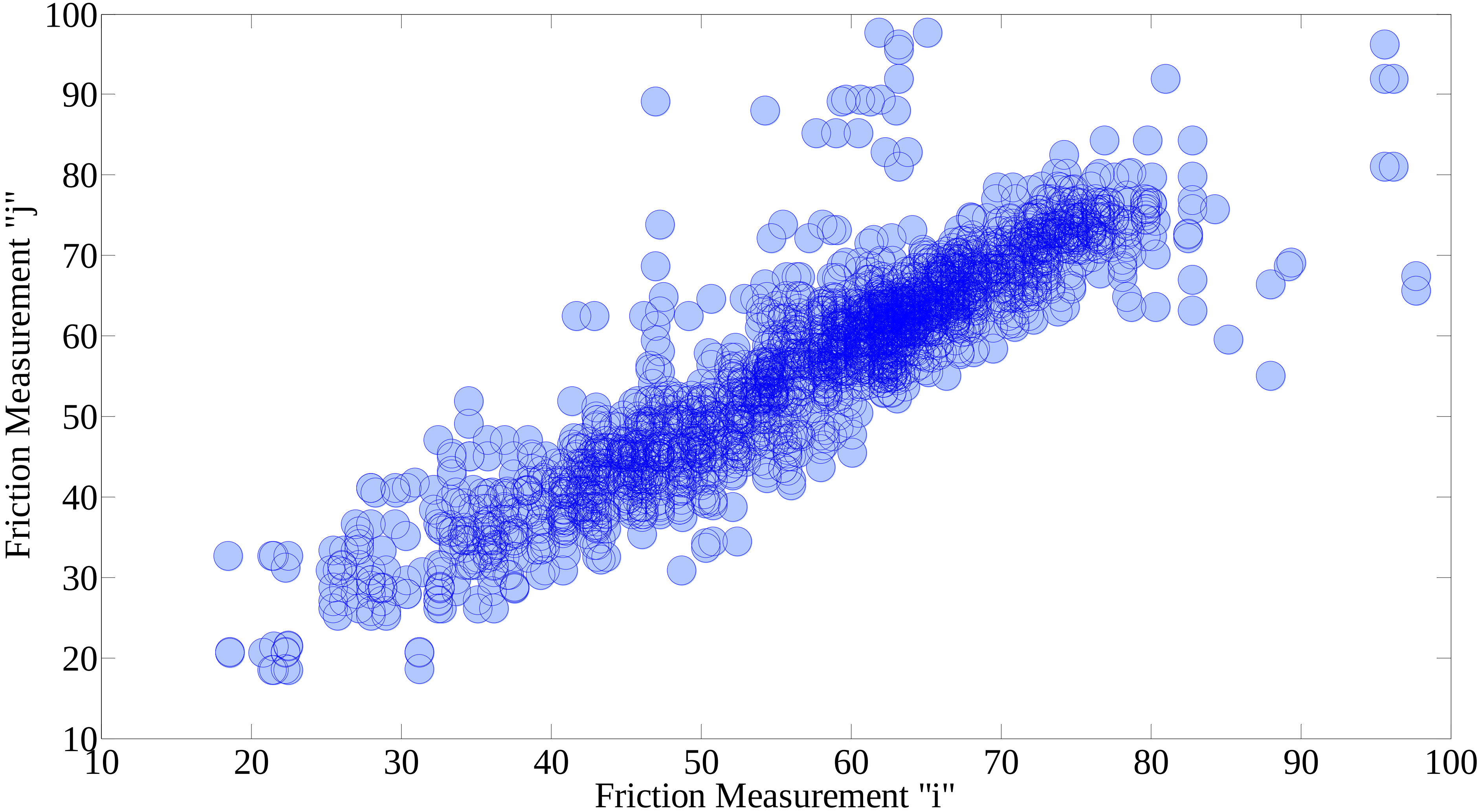
$$s_{\zeta}^{2} = \mathrm{var}(\zeta)$$

$$\lambda = \frac{\tau_{Y}^{2} + \sigma_{Y}^{2}}{\tau_{Z}^{2} + A\sigma_{Z}^{2}}$$

- **τ = repeatability error standard deviation**
- **σ = model error standard deviation**
- **Solution is found iteratively**

Practical Results

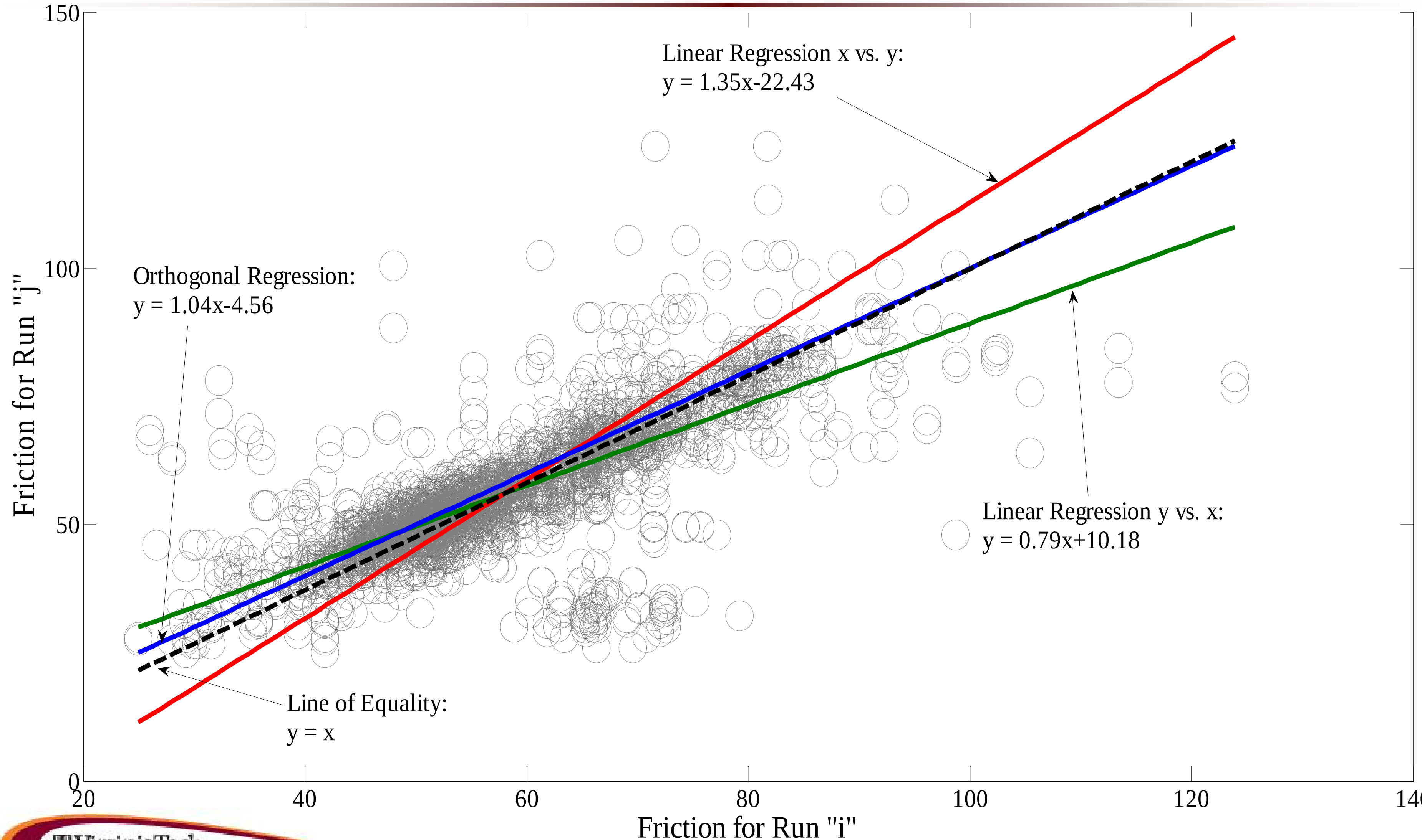
Measurements from Same Device



Model Parameters

- True parameters:
  - ✓ Slope = 1
  - ✓ Intercept = 0
- Ordinary regression
  - ✓ Slope = 0.9262
  - ✓ Intercept = 3.3002
- Measurement error model:
  - ✓ Slope = 1.0207
  - ✓ Intercept = -2.1425

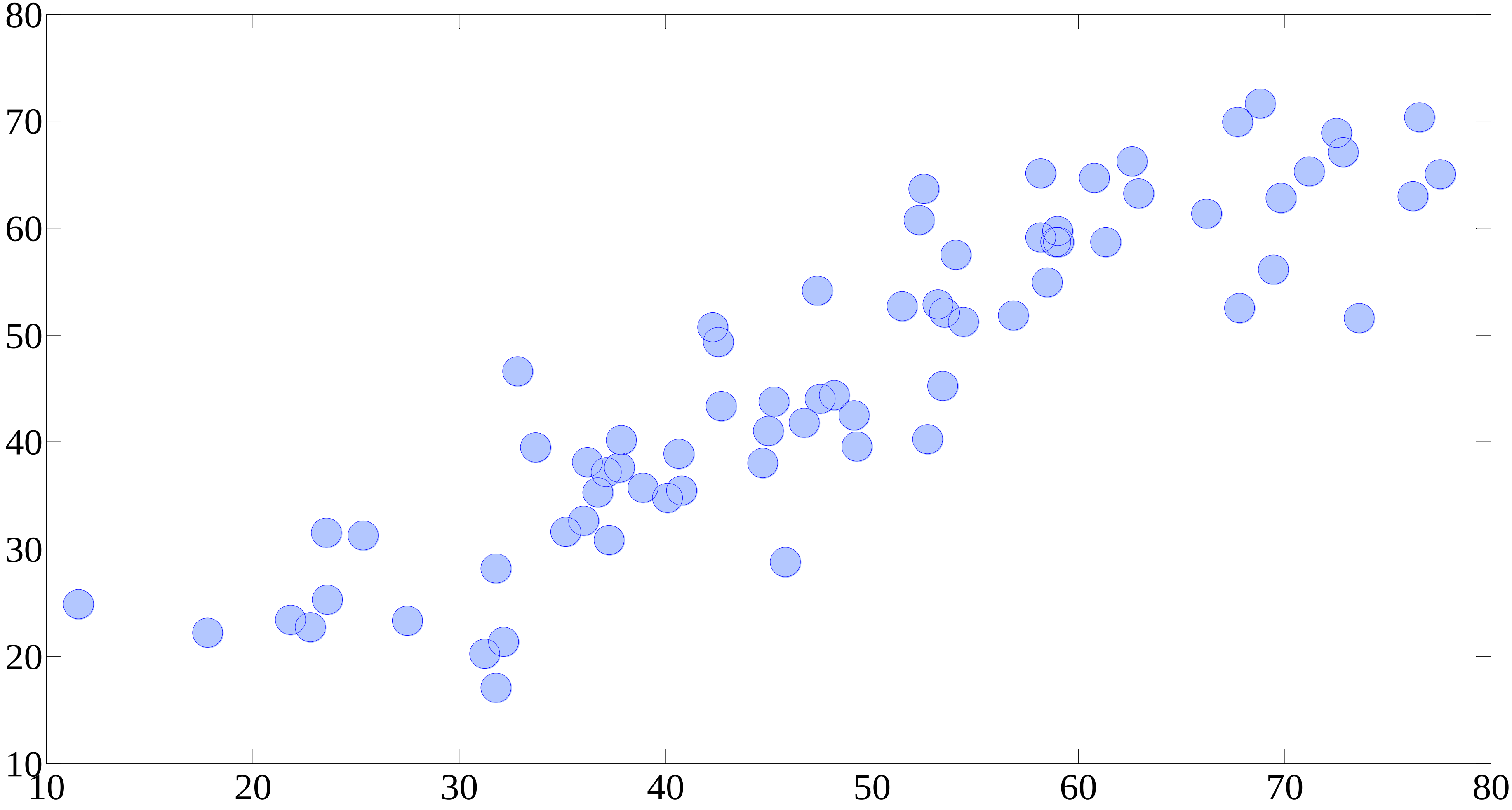
Dangers of Speed and Texture Correction



Model Parameters

- True parameters:
  - ✓ Slope = 1
  - ✓ Intercept = 0
- Ordinary regression
  - ✓ Slope = 0.7905 (previous 0.9262)
  - ✓ Intercept = 10.1880 (previous 3.3002)
- Measurement error model:
  - ✓ Slope = 1.0448 (previous 1.0207)
  - ✓ Intercept = -4.5637 (previous -2.1425)

Different Devices (Simulation)



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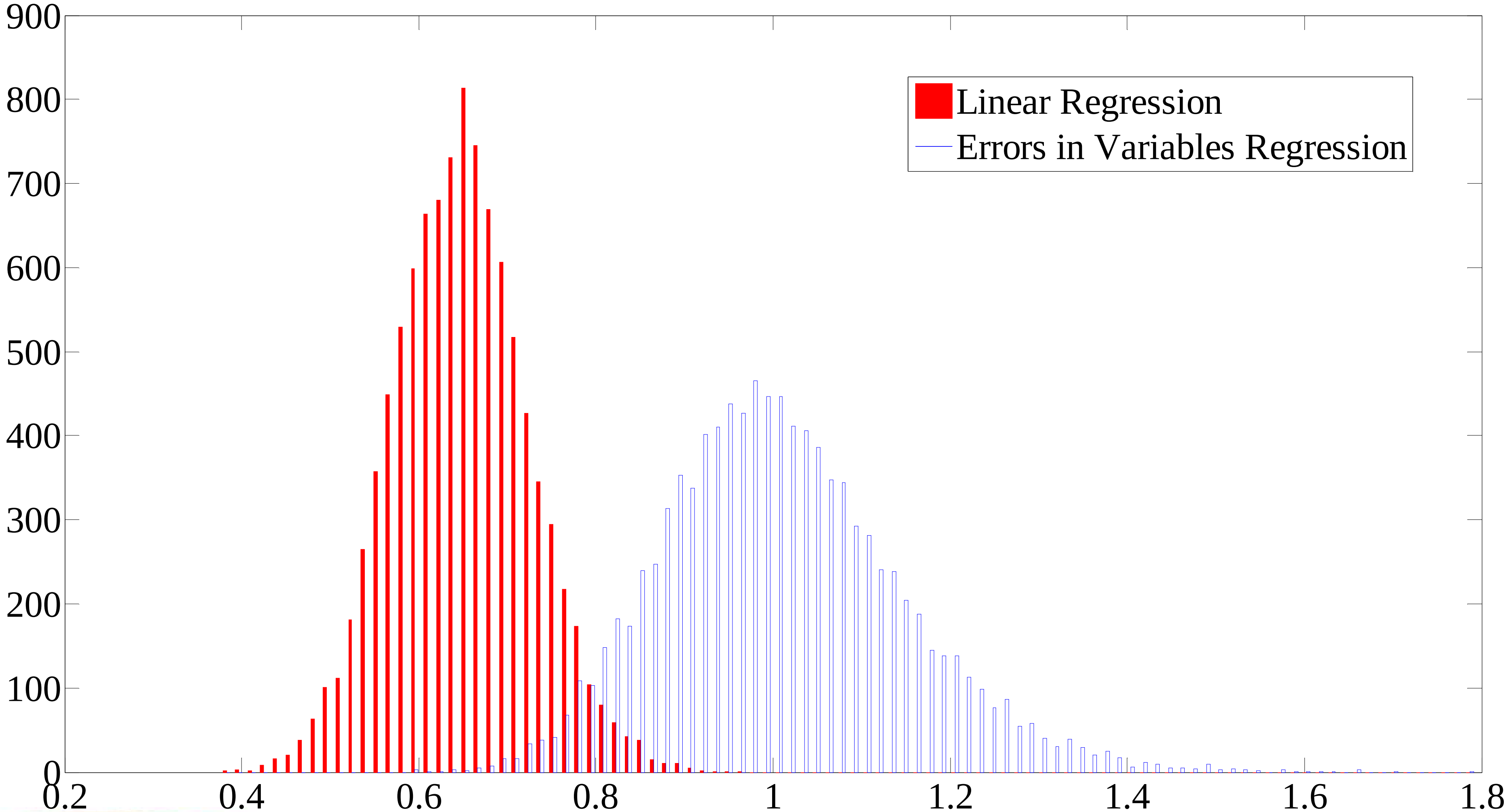

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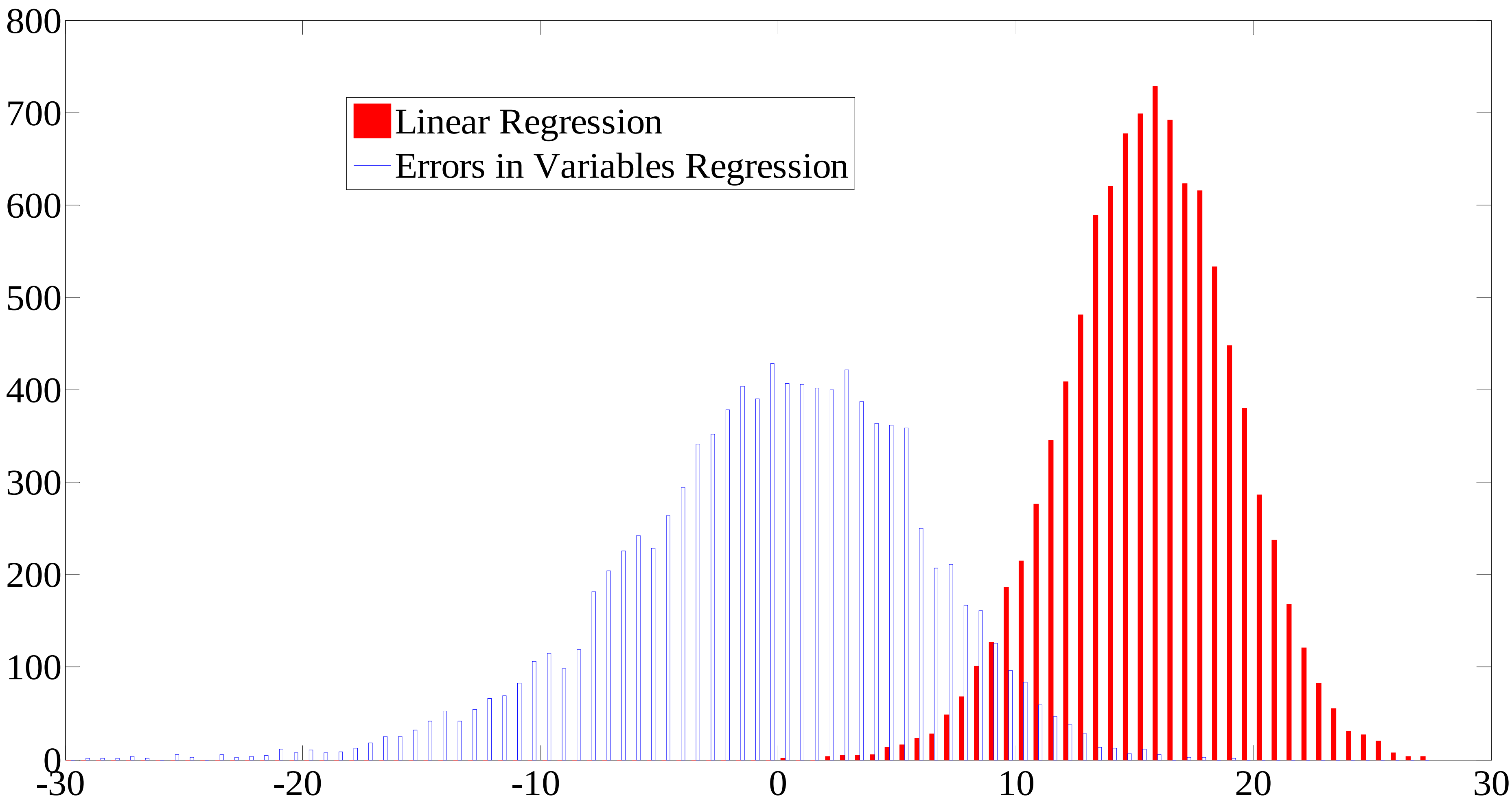

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Different Devices (Simulation)

$$Y = 1.5X + 20 + \varepsilon_Y \qquad Z = 0.75X - 10 + \varepsilon_Z$$

- **True parameters:**

$$A = 1.5/0.75 = 2 \qquad B = 20 - 1.5/0.75\left(-10\right) = 40$$

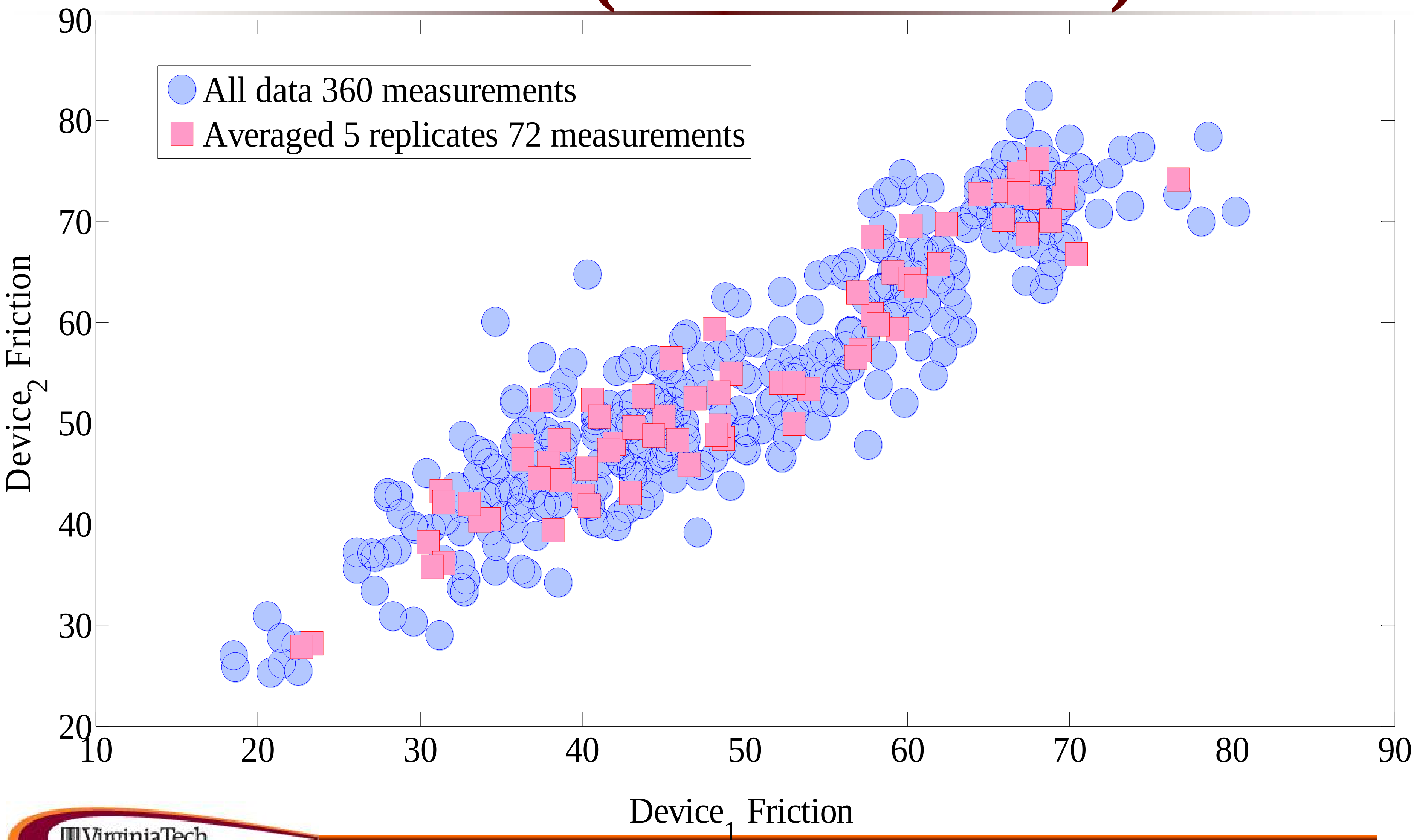
- **Ordinary regression**

$$E\left\{\hat{A}\right\} = 1.2066 \qquad E\left\{\hat{B}\right\} = 57.5459$$

- **Measurement error model:**

$$E\left\{\hat{A}\right\} = 2.0255 \qquad E\left\{\hat{B}\right\} = 39.3649$$

Measurements from Different Devices (Locked Wheel)



Model Parameters

- **Averaged:**
  - ✓ **Ordinary regression:**
    - ➢ **Slope = 0.8927; Intercept = 10.3757**
  - ✓ **Measurement error model:**
    - ➢ **Slope = 0.9341;Intercept = 8.3281**
- **Not Averaged:**
  - ✓ **Ordinary regression:**
    - ➢ **Slope = 0.8590 (bias = 0.8927-0.8590 = 0.0337)**
  - ✓ **Measurement error model:**
    - ➢ **Slope = 0.9346 (bias = 0.9341-0.9346 = -0.0005)**

Limits of Agreement (LOA)

- **Relationship between devices:**
  - ✓ **Convert between measurements**
  - ✓ **Does NOT give agreement**
- **After conversion:**
  - ✓ **Limits of Agreement (LOA)**
  - ✓ **General: Bland and Altman (1983), Altman and Bland (1986)**
  - ✓ **Friction devices: De Leon et al. (2012)**

Conclusions

- **Ordinary regression:**
  - ✓ **Biased estimate of the relationship**
  - ✓ **Inconsistent relationship based on choice of dependent and independent variable**
- **Measurement Error Model:**
  - ✓ **Unbiased estimate of the relationship**
  - ✓ **Relationship independent of choice of dependent and independent variables**

Conclusion

- **ASTM**
  - ✓ **“the adjustment of the outputs of different devices used for the measurement of a specific phenomenon so that all devices report the same value”**
- **Ordinary regression**
  - ✓ **The same device WILL NOT report the same value**

Reproducible Research

- Paper has been submitted to TRB
- Paper written under the reproducible research paradigm:
  - ✓ All data used in the paper is made available for anybody to use
  - ✓ Matlab files used are made available:
    - Calculations organized in functions
    - GUI that can be used to reproduce all Figures showing the results

PAVEMENT EVALUATION 2014

SEPT. 14-18, 2014

Originally held in 2002, this conference builds on the previous conference's accomplishments by combining the annual Road Profiler Users' Group meeting with discussions and presentations from other disciplines of non-destructive pavement evaluation. In addition to profiling, other topics of interest include texture and friction measurement, tire-pavement noise, ground penetrating radar (GPR), video distress rating and structural testing.The conference welcomes representatives from government transportation agencies, academia and private industry. It will benefit end-users, operators, researchers, construction and design engineers, and manufacturers who have an interest in the equipment, methods, and use of non-destructive pavement evaluation. It is also highly recommended for consultants, contractors, and construction equipment developers who are in the business of meeting performance requirements for traveled surfaces.

After the meeting, equipment owners and manufacturers will be invited to participate in an equipment comparison rodeo to be conducted at the Virginia Smart Road in Blacksburg, Virginia.

Blacksburg, VA

RPUG

VirginiaTech Transportation Institute

U.S. Department of Transportation Federal Highway Administration

VTRC Virginia Transportation Research Council

VDOT Virginia Department of Transportation

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